Addition – progression in written methods Y1 to Y6

$$1 + 5 = 6$$

addend   addend   sum

Contextualise the mathematics
- **WHAT DOES THIS NUMBER REPRESENT?**

Expose mathematical structure and work systematically

Expect children to use correct terminology and express reasoning
- Use **STEM SENTENCES**
- Answer in **complete sentences**

Identify difficult points
- Be aware of common misconceptions
- Actively seek to uncover these

Move between the concrete, pictorial and the abstract (CPA)

Teach inequality alongside equality

- < and > can also help deepen understanding of key concepts,
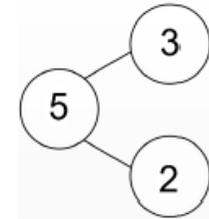 eg   18p   £0.15

Use empty box problems
- Promotes reasoning and finding easy ways to calculate
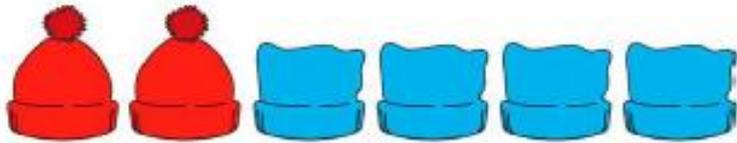- Use a sequence to develop conceptual connections

ADDITION – YEAR 1

Use ten frames to regroup numbers to make 10.

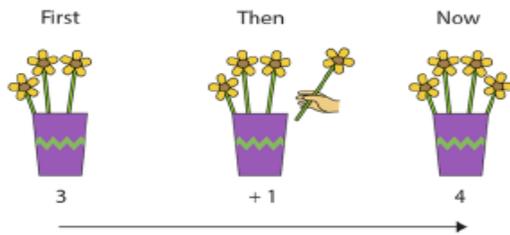Use part-whole diagram (include zero).

Zero is not a part.

5 — 3
  — 2

Start with expressions with no equal sign (=)   Later, move on to equations (has = sign)

2 + 4
4 + 2

5 = 3 + 2

First   Then   Now   Representations

3   +1   4

First   Then   Now

6   +1   7

5	

3 + 1 = 4   6 + 1 = 7   3 + 2 = 5

ADDITION – YEAR 2

£16 +
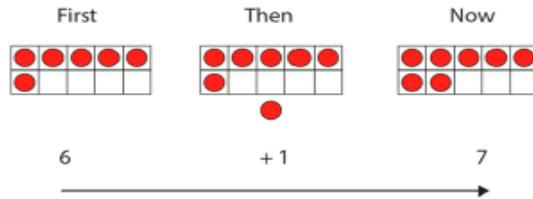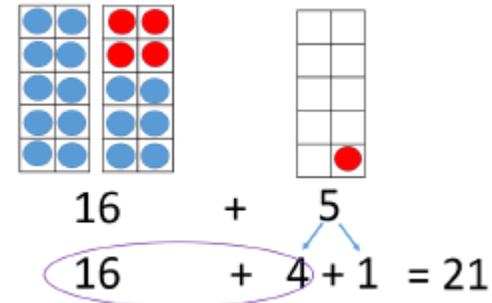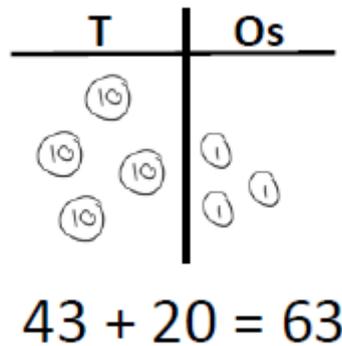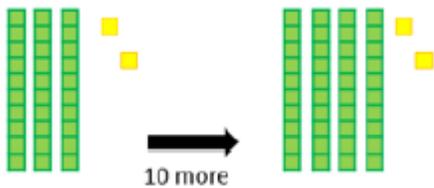
At **first** Fiona had saved £34 and **then** she added her £3 pocket money to that.

How much does she have **now**?
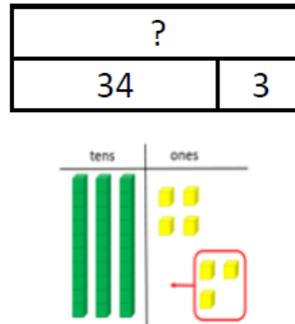
?	
34	3

tens | ones

16 + 5

16 + 4 + 1 = 21

10 more

T | Os

$43 + 20 = 63$

$27 + 14$

$27 + 10 + 4$

$37 + 4 = 41$

ADDITION – YEAR 3

2 dn + 2dn- Keep the first number whole.

T | Os

64 + 30 = 94

27 + 14

(27 + 10) + 4

(37) + 4 = 41

$$\begin{array}{r} 23 \\ + 14 \\ \hline 37 \end{array}$$

2dn + 1dn

26 + 5
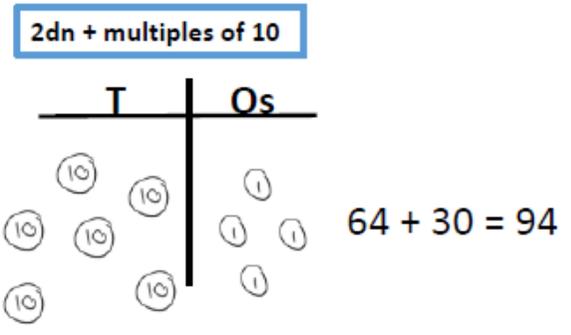
26 + 4 + 1 = 31

Column Method

Unitise : e.g. 448 + 214

- **8 ones + 4 ones equals 12 ones. We rename this: it is one ten and 2 ones.**
- **4 tens add one ten add one 10 equals 6 tens.**
- **4 hundreds add 2 hundreds equals 6 hundreds.**

$$\begin{array}{r} 48 \\ + 14 \\ \hline 62 \\ \hline 1 \end{array}$$

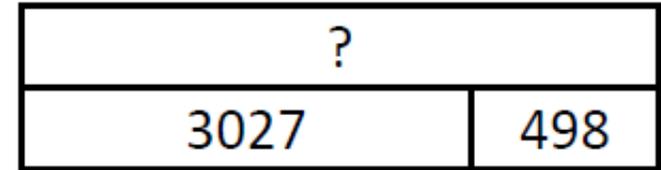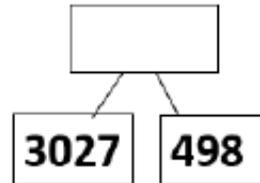$$\begin{array}{r} 258 \\ + 165 \\ \hline 423 \\ \hline 1\ 1 \end{array}$$

exchanged figure at the bottom

$$\begin{array}{r} 58 \\ + 1\blacksquare \\ \hline 75 \end{array}$$

solve missing box problems

ADDITION – YEAR 4

| 3027 | 498 |

?	
3027	498

Column Method

Unitise : e.g. 1418 + 294

- 8 ones + 4 ones equals 12 ones. We rename this: it is one ten and 2 ones.
- 1 ten add 9 tens add 1 ten equals 11 tens. We rename this: it is one hundred and 1 ten. Move away from 10 + 90 + 10.
- 4 hundreds add 2 hundreds add 1 hundred equals 7 hundreds.
- One thousand add no thousands equals one thousand.

Solve missing box problems

```
  7 5 8
+ ■ 1 5
-------
1 0 ■ 3
```

```
  258
+ 165
-----
  423
   1 1
```

```
  7289
+ 5145
------
 12434
  1  1 1
```

ADDITION – YEAR 5

?	
375.5	14.3

Problem solving

Amy and Matthew are playing their favourite
computer game. Amy's current high
score is 8,524.
Matthew's high score is bigger than
Amy's and when
you add them together their combined
total is 19,384.
What is Matthew's high score?

Work out the missing numbers.

$$\begin{array}{r} \blacksquare 4 \blacksquare 3 \blacksquare \\ + \ 2 \blacksquare 5 \blacksquare 2 \\ \hline 78529 \end{array}$$
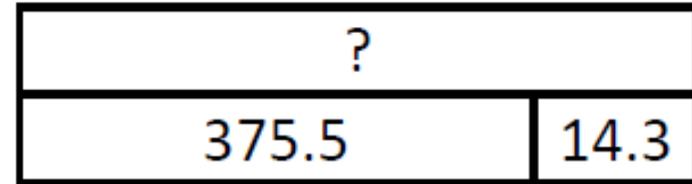
Column Method

Unitise : e.g. 1418 + 294

- 8 ones + 4 ones equals 12 ones. We rename this: it is one ten and 2 ones.
- 1 ten add 9 tens add 1 ten equals 11 tens. We rename this: it is one hundred and 1 ten. Move away from 10 + 90 + 10.
- 4 hundreds add 2 hundreds add 1 hundred equals 7 hundreds.
- One thousand add no thousands equals one thousand.

- Vary the number of digits in the number.
- Put = sign on right hand side
- Balanced equations
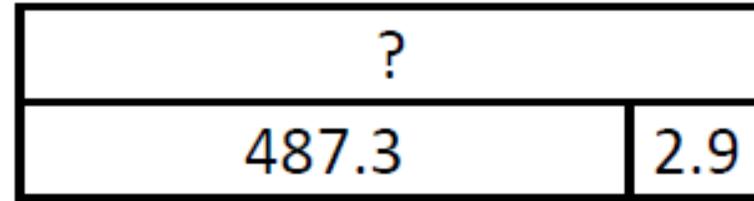
65 + 577 =

? = 4277 + 656

648 + ? = 1036 + 58

$$\begin{array}{r} 57.30 \\ + \ \ 6.08 \\ \hline 63.38 \end{array}$$

1

Decimal addition with different number of digits.

ADDITION – YEAR 6

?	
487.3	2.9

- **Vary the number of digits in the number.**
- **Put = sign on right hand side**
- **Balanced equations**

65 + 577 =

? = 4277 + 656

648 + ? = 1036 + 58

Problem solving

A is an odd number which rounds to 100,000 to the nearest ten thousand.
It has a digit total of 30.
B is an even number which rounds to 500,000 to the nearest hundred thousand.
It has a digit total of 10.
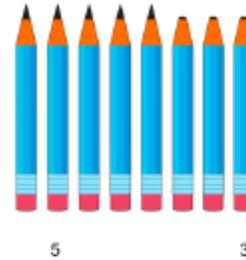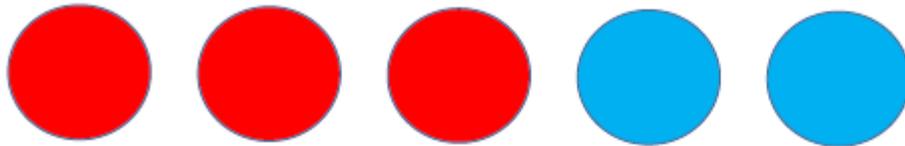A and B are both multiples of 5 but end in different digits.

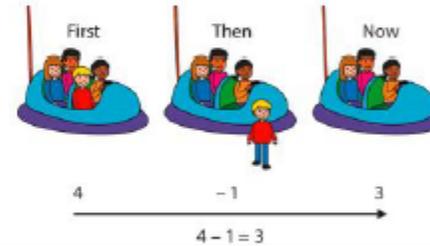| A	B
631,255	

Subtraction – progression in written methods Y1 to Y6

$$8 - 1 = 7$$

minuend   subtrahend   difference

Partitioning – the 'not' structure
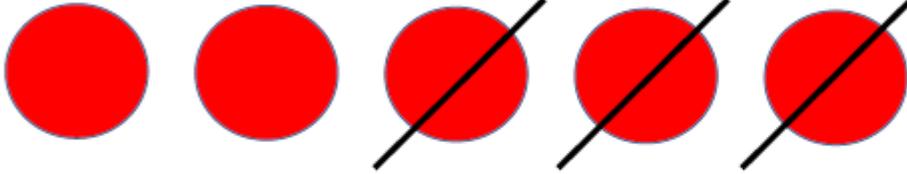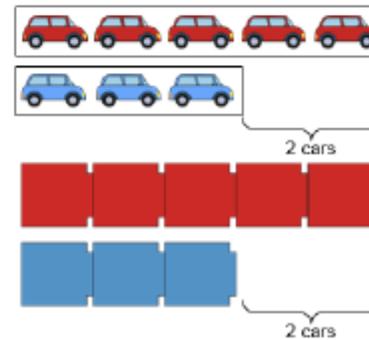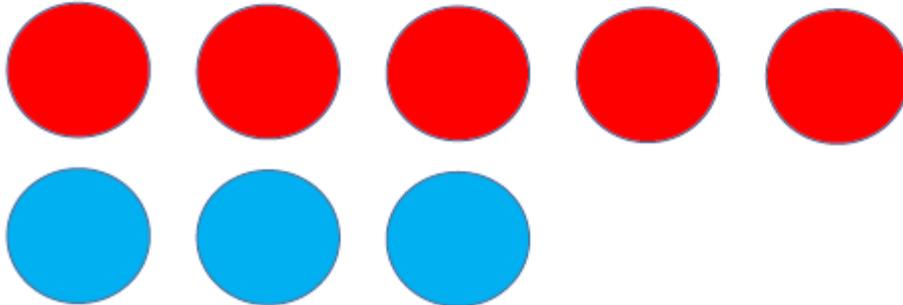
5   3

Reduction

First   Then   Now

4   – 1   3

$4 - 1 = 3$

Difference

2 cars
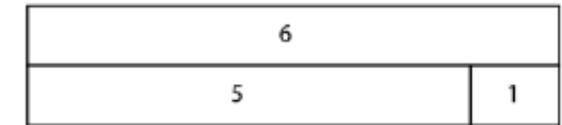
2 cars

SUBTRACTION – YEAR 1

Start with expressions with no equal sign (=)

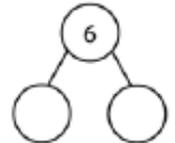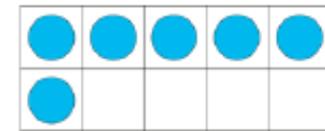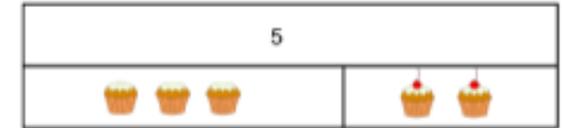**Later, move on to equations (has = sign)**

8 - 3

Partitioning

Difference

Reduction

2 dn - 1 dn Use numbers in context.

What does each number represent?

5	

6	
5	1

First   Then   Now

3   – 0   3

3 – 0 = 3

First	Then	Now

☐ - ☐ ☐

☐ - ☐ = ☐

difference of 4
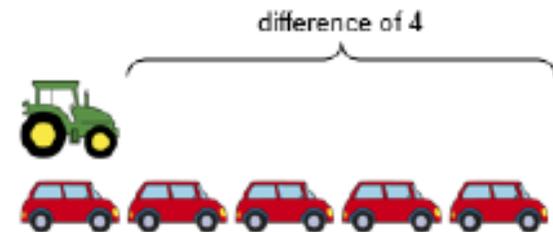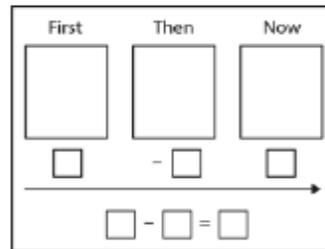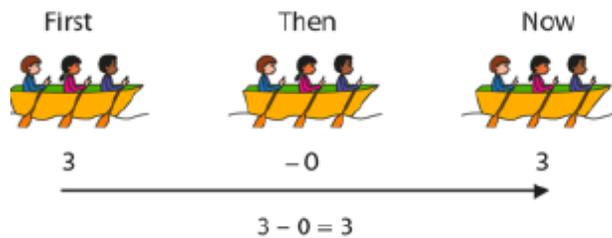
SUBTRACTION – YEAR 2

At **first** Fiona had £24 and **then** she spent £5.

How much does she have **now**?

£24 -

24 - 5

24 - 4 - 1 = 19
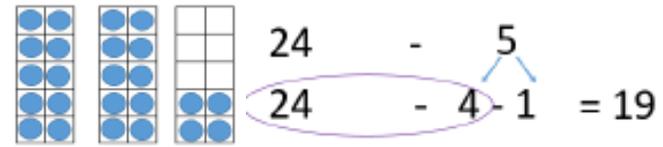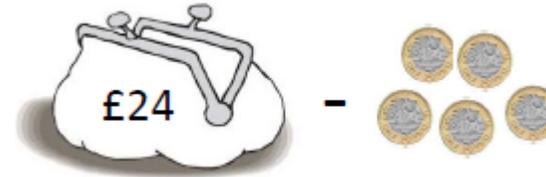
Use the part whole model and bar model

10 less

2 dn – 2 dn

Keep the first number whole.

24

?

5

T	Os
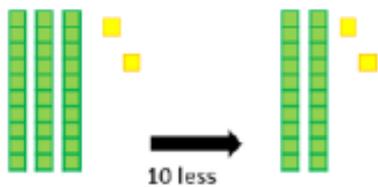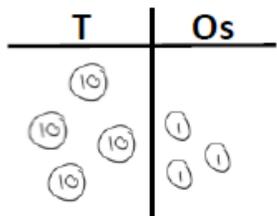
43 - 20 = 23

58 - 17

58 - 10 - 7

48 - 7 = 41
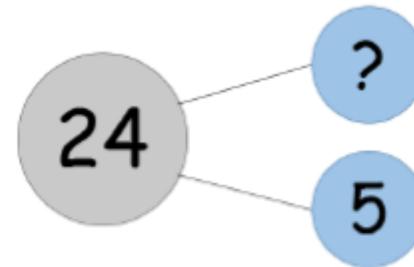
24	
19	5

SUBTRACTION – YEAR 3

Subtraction is not commutative.

Pupils need to practice mental strategies as well as formal methods.e.g.

Count On if the numbers are close together $203 - 199$

Count Back if subtracting a single digit or multiple of ten $324 - 5$ or $257 - 40$

Round and adjust if subtracting a near tens number $257 - 19$

Problem solve using written method

Some numbers are given.

| 1 | 3 | 4 | 5 | 7 | 8 |

Use the numbers to form two 3-digit numbers.

Subtract the numbers to get the greatest answer.

Show your work on

h	t	o

2 dn - 2 dn - Keep 1st number whole.

58 - 17

58 - 10 - 7

48 - 7 = 41

T	Os
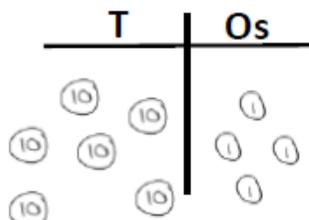
64 - 30 = 34

3 dn - 3 dn
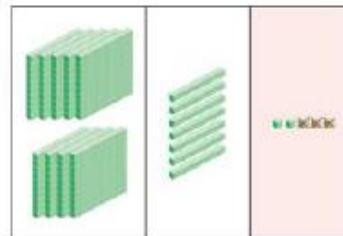
Column method- unitise the values e.g. 5 ones subtract 3 ones equal 2 ones. 7 tens subtract 2 tens equal 5 tens etc.

Formal colum method. Children must understand what has happened when they have crossed out digits.

h	t	o
9	7	5
- 7	2	3
		2

234
- 88
6

SUBTRACTION – YEAR 4

Column method- unitise the values

13 ones subtract 8 ones equal 5 ones. 3 tens subtract 1 ten equal 2 tens etc.

Children to use the bar model and part whole model to develop estimation and number sense.

$$\begin{array}{r} 4\overset{3}{\cancel{4}}\overset{1}{3} \\ -218 \\ \hline 225 \end{array}$$

hundreds	tens	ones

$443 - 218 = 225$

3027

? | 498

3027	
?	498
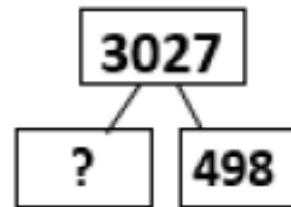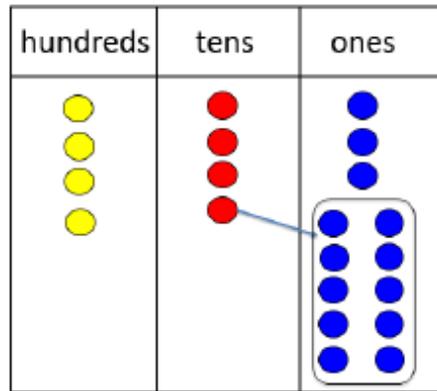
Problem Solving

Can you complete the wall?

495

269

121

76

Missing digit calculations

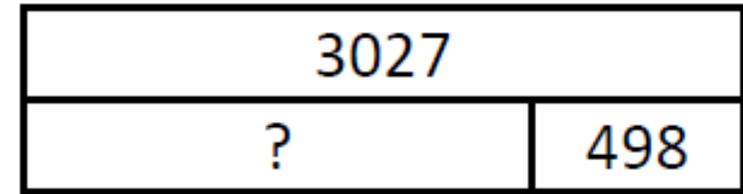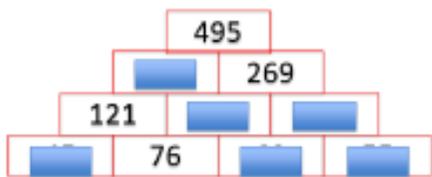$$\begin{array}{r} 3 \quad 9 \quad \square \\ - \square \square \square \quad 6 \\ \hline \square \quad 0 \quad 5 \end{array}$$

$5\square 28 - 44\square = 4788$

$\square\square\square 0 - 2468 = 5092$

Write >, = or < in each of the circles to make the number sentence correct.

$1023 + 24 + 24 \bigcirc 1023 + 48$

$1232 - 232 \bigcirc 1355 - 252$

$1237 - 68 + 32 \bigcirc 1242 - 69 + 31$

SUBTRACTION – YEAR 5

Select an appropriate method.

Column method

Unitise: 8 tenths subtract 5 tenths.

375.5	
?	14.3

$$\begin{array}{r} \overset{6}{\cancel{7}}\overset{1}{1}.8 \\ -\ \ 34.5 \\ \hline 37.2 \\ \hline \end{array}$$

Problem Solving

What is the total cost for a return journey to York for one adult and two children? How much more does it cost for two adults to make a single journey to Hull than to Leeds?

		York	Hull	Leeds
Adult	Single	£13·50	£16·60	£11·00
	Return	£24·50	£30·00	£20·00
Child	Single	£9·75	£11·00	£8·00
	Return	£15·00	£18·50	£13·50

Work out whether each problem is true or false and say how he could solve the problem if it is wrong.

a)   3801 + 1499 = 3800 + 1500

b)   3801 + 2307 = 3800 + 2310

c)   5678 − 1212 = 5670 − 1220

d)   5678 − 152 = 5676 − 150

SUBTRACTION – YEAR 6

M

487.3	
?	2.9

$15.743 - 214.9 =$

$? - 200 = 2,307$

$$\frac{5}{6} - \frac{1}{4} =$$

$$
\begin{array}{r}
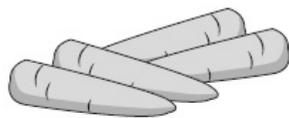\overset{2\ 1}{57.\cancel{3}0} \\
-\ \ 6.08 \\
\hline
51.22
\end{array}
$$

$$
\begin{array}{r}
2\ 9\ \overset{1}{\cancel{2}}\ 1 \\
3\ \cancel{0}\ \cancel{3}\ 1\ .\ 8 \\
-\ 1\ 8\ 6\ 7\ .\ 3 \\
\hline
1\ 1\ 6\ 4\ .\ 5
\end{array}
$$

potatoes
£1.50 per kg

carrots
£1.80 per kg

Jack buys $1\frac{1}{2}$ kg of potatoes and $\frac{1}{2}$ kg of carrots.

How much **change** does he get from **£5**?

There are 2,400 leaflets in a box.

William and Ally take 450 leaflets each.

Adam and Chen share the rest of the leaflets equally.

How many leaflets does Adam get?

Multiplication Written Calculation Progression Year 1 to Year 6

Vocabulary: product, factor, multiple, 2 x3 is 2, 3 times, lots of, groups of, repeated addition, multiplier, multiplicand
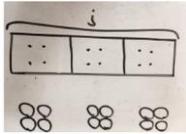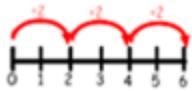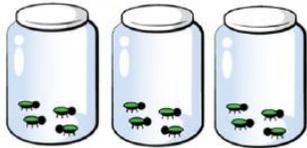
Year 1
- Use concrete objects to show repeated addition.
- Commutative property.

Context driven

Concrete   Pictorial

There are 3 equal groups, with 4 in each group.
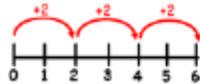
Year 2
- 2, 5, 10 times tables
- Use concrete objects to show repeated addition.
- Arrays- commutative properties

Context driven

Concrete   Pictorial   Abstract

2 x 5 = 10

5 x 2 = 10

Year 3
- 3, 4 and 8 times tables
- 2 digit x 1 digit
- Arrays- commutative properties
- Bar model to show the multiplicand and repeated addition.

HORIZONTAL PARTITIONING

36 x 4

6 x 4 =   24

30 x 4 = 120

144

4 x 8 = 32

8 x 4 = 32

Year 4
- Multiplication 12 x 12
- Multiply by 0 and 1
- Commutative property
- Distributive property 6 x (3+1) = 6 x 3 + 6 x 1

HORIZONTAL PARTITIONING
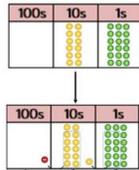
78 x 4

8 x 4 =   32 +

70 x 4 = 280

312

EXPANDED COLUMN

(Short time only)

7 8

x 4

32 (8 x 4)

280 (70 x 4)

312

Formal column method with place value counters.
6 x 23

6 x 23 =

23

x 6

138

Year 5
- As for Year 4
- 1 digit x 1 digit x 1 digit
- 3 digit x 1 digit
- 4 digit x 2 digit
- Use bar model to show repeated addition.

1 2 4

x   2 6

7 4 4

2 4 8 0

3 2 2 4

Multiplicand   '4' is the Multiplier

Year 6
- As for Year 5
- Multiply decimals
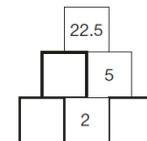- Commutative and distributive properties

Bar model good for estimation

1 2 4

x   2 6

7 4 4

2 4 8 0

3 2 2 4

Answer: 3224

Here is a number pyramid.

The number in a box is the **product** of the two numbers below it.

Write the missing numbers.

22.5

5

2

Division- Written Calculation Progression from Year 1 to Year 6

grouping, sharing, equal groups, dividend, divisor, quotient, factor, remainder

Yr 1
- Use concrete objects
- Contexts- sharing and grouping

Context based

Concrete → Pictorial

Link to fractions 1/2 of ... and ÷ 2

Yr 2
- Solve problems involving division.
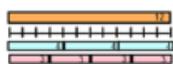- Not commutative
- Grouping and sharing contexts
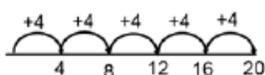
Context based

Concrete → Pictorial → Abstract

Link to fractions 1/4 of ... and ÷ 4   20 ÷ 4 = 5

Arrays are useful to show how division and multiplication are linked

Bar models also show the connection between division and multiplication

+4 +4 +4 +4 +4   Number lines are good to show grouping
4   8   12 16 20

Yr 3
- 2 digit ÷ 1 digit
- Grouping and sharing contexts

Context based

Concrete → Pictorial → Abstract

Link to fractions 1/3 of ... and ÷ 3   18 ÷ 3 = 6

42 ÷ 3
Start with the biggest place value.
We are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.
We exchange this ten for 10 ones and then share the ones equally among the groups.
We look at how many are in each group.

Yr 4
- Up to 3 digits ÷ 1 digit
- Short division as a written method.

Context - grouping and sharing problems

Concrete → Pictorial → Abstract

Link to fractions 1/6 of ... and ÷ 6   126 ÷ 6 = 21

132 ÷ 3   3 [132   ]
                    120
            40 lots of 3

The bar model is good for developing number sense and estimation.

   4 4
3) 1 3¹2   Short division (bus stop) method

Yr 5
- Up to 4 digits ÷ 1 digit
- Short division
- Interpret the remainder

Context - grouping and sharing problems

Link to fractions 1/7 of ... and ÷ 7   1426 ÷ 7 = 203 r5

432 ÷ 5   5 [432]   What is 500 divided by 5?

Draw the bar model to estimate and discuss reasonable answers.

      8 6 r 2   Interpret the remainder: use a context to decide how to interpret it
5) 4 3³2

Yr 6
- Up to 4 digits ÷ 2 digits
- Short division or long division
- Division using factors
- Interpret remainder

Context - grouping and sharing problems

Link to fractions 1/7 of ... and ÷ 7   1426 ÷ 7 = 203 r5

1426 ÷ 7   7 [1426]   What if the calculation was 1400 divided by 7?

Draw the bar model to estimate and discuss reasonable answers.

       2 8 . 8   Use times table ladders to help estimation
15) 4 3 2 . 0
     3 0 | 150 (15 x 10)
     1 3 2
     1 2 0 |
       1 2 0   75 (15 x 5)
       1 2 0
       0 0 0   15 (15 x 1)